Seven dimensions of the world

Fundamental quantities	Dimensions
Length	[L]
Mass	[M]
Time	[T]
Temperature	[K]
Current	[A]
Amount of substance	[N]
Luminous intensity	[J]

Dimensions of a physical quantity

The powers of fundamental quantities in a derived quantity are called dimensions of that quantity.

Dimensions of a physical quantity

Example:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{\text{Mass}}{\text{length} \times \text{breath} \times \text{height}}$$

$$[\text{Density}] = \frac{[M]}{[L] \times [L] \times [L]} = \frac{[M]}{[L^3]} = [ML^{-3}]$$

Hence the dimensions of density are 1 in mass and -3 in length.

Uses of Dimension

To check the correctness of equation

To convert units

To derive a formula

To check the correctness of equation

Consider the equation of displacement,

$$\Delta x = v_i t + \frac{1}{2} a\, t^2$$

By writing the dimensions we get,

$$\Delta x = \text{displacement} = [L]$$

$$v_i t = \text{velocity} \times \text{time} = \frac{\text{length}}{\text{time}} \times \text{time} = [L]$$

$$a t^2 = \text{acceleration} \times \text{time}^2 = \frac{\text{length}}{\text{time}^2} \times \text{time}^2 = [L]$$

The dimensions of each term are same, hence the equation is dimensionally correct.

To convert units

Let us convert newton (SI unit of force) into dyne (CGS unit of force).

The dimesions of force are = $[LMT^{-2}]$

So, $1 \text{ newton} = (1\text{ m})(1\text{ kg})(1\text{ s})^{-2}$

and, $1 \text{ dyne} = (1\text{ cm})(1\text{ g})(1\text{ s})^{-2}$

Thus, $$\frac{1 \text{ newton}}{1 \text{ dyne}} = \left(\frac{1\text{ m}}{1\text{ cm}}\right)\left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2} = \left(\frac{100\text{ cm}}{1\text{ cm}}\right)\left(\frac{1000\text{ g}}{1\text{ g}}\right)\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= 100 \times 1000 = 10^5$$

Therefore, $1 \text{ newton} = 10^5 \text{ dyne}$

S. No.	Physical quantity
1.	Area
2.	Volume
3.	Density=mass/volume
4.	Speed or velocity $= \frac{dx}{dt}$
5.	Acceleration $= \frac{dv}{dt}$
6.	Force=mdv/dt=ma
7.	Linear Momentum (p=mv)
8.	Impulse $= F.\Delta t$
9.	Power=work/time
10.	Work or Energy =force× displacement
11.	Pressure or Stress =force/area

S. No.	Physical quantity	Dimensional formula	M.K.S. units
1.	Area	$\left[M^0L^2T^0\right]$	m^2
2.	Volume	$\left[M^0L^3T^0\right]$	m^3
3.	Density=mass/volume	$\left[ML^{-3}T^0\right]$	$kg\ m^{-3}$
4.	Speed or velocity $=\frac{dx}{dt}$	$\left[M^0LT^{-1}\right]$	$m\ s^{-1}$
5.	Acceleration $=\frac{dv}{dt}$	$\left[M^0LT^{-2}\right]$	$m\ s^{-2}$
6.	Force=mdv/dt=ma	$\left[MLT^{-2}\right]$	$kg\ m\ s^{-2}$ or N
7.	Linear Momentum (p=mv)	$\left[MLT^{-1}\right]$	$kg\ m\ s^{-1}$ or Ns
8.	Impulse $= F.\Delta t$	$\left[MLT^{-1}\right]$	$kg\ m\ s^{-1}$ or $N\ s$
9.	Power=work/time	$\left[ML^2T^{-3}\right]$	$J\ sec^{-1}$ or watt or $kg\ m^2\ s^{-3}$
10.	Work or Energy =force× displacement	$\left[ML^2T^{-2}\right]$	$N\ m$ or J or $kg\ m^2\ s^{-2}$ or $g\ cm^2\ s^{-2}$
11.	Pressure or Stress =force/area	$\left[ML^{-1}T^{-2}\right]$	$N\ m^{-2}$ or $kg\ m^{-1}\ s^{-2}$

To derive a formula

The time period 'T' of oscillation of a simple pendulum depends on length '*l*' and acceleration due to gravity '*g*'.

Let us assume that,

$$T \propto l^a g^b \qquad \text{or} \qquad T = K\, l^a g^b$$

K = constant which is dimensionless

Dimensions of T = $[L^0M^0T^1]$

Dimensions of l = $[L^1M^0T^0]$

Dimensions of g = $[L^1M^0T^{-2}]$

Thus, $[L^0M^0T^1] = K\,[L^1M^0T^0]^a\,[L^1M^0T^{-2}]^b$

$$= K\,[L^aM^0T^0]\,[L^bM^0T^{-2b}]$$

$$[L^0M^0T^1] = K\,[L^{a+b}M^0T^{-2b}]$$

$$a + b = 0 \qquad \& \qquad -2b = 1$$

$$\therefore \quad b = -\frac{1}{2} \qquad \& \qquad a = \frac{1}{2}$$

$$T = K\, l^{1/2} g^{-1/2}$$

$$\therefore \quad T = K\sqrt{\frac{l}{g}}$$

Least count of instruments

The smallest value that can be measured by the measuring instrument is called its least count or resolution.

LC of length measuring instruments

Ruler scale

Least count = 1 mm

Vernier Calliper

Least count = 0.1 mm

LC of length measuring instruments

Screw Gauge

Least count = 0.01 mm

Spherometer

Least count = 0.001 mm

LC of mass measuring instruments

Weighing scale

Least count = 1 kg

Electronic balance

Least count = 1 g

LC of time measuring instruments

Wrist watch

Least count = 1 s

Stopwatch

Least count = 0.01 s